Talk at NEU student seminar April 2013

„Infinitesimal Cherednik Algebras"

<u>Plan</u>:

0. History: Drinfeld '86 $\longrightarrow$ EG '02 $\longrightarrow$ EGG '05 $\longrightarrow$ Tik '08-...
1. Explain the basic things about continuous Hecke alg's
   $\hookrightarrow$ definition, examples as cont. SRA and cont. Cher. alg.
2. Infinitesimal Cher. alg-s following [EGG]. Mention cat. $\mathcal{O}$ for $\mathfrak{gl}_n$ ([DT])
3. Finite W alg-s
4. Main thm.
   Consequences: Center, finitely many sympl. leaves
   Completions: ...

① In 2005, Etingof-Gan-Ginzburg defined the so-called continuous Hecke algebras, which are "continuous analogues" of the Drinfeld's degenerate affine Hecke algebras, particular case of which was rediscovered by Etingof-Ginzburg under the name of SRA (sympl. refl-n alg-s)

$\boxed{\text{Continuous Hecke algebras}}$

$\cup$ $\boxed{\text{continuous sympl. reflection algebras}}$

$\cup$ $\boxed{\text{continuous Cherednik algebras}}$ ← This is the topic of today's talk

Motivation for studying them: their representation theory unifies RT of real reductive gps, SRA, Drinfeld-Lusztig degenerate affine Hecke algebras.

② • G - reductive algebraic group
• $\rho : G \to GL(V)$ - algebraic representation
• $\mathcal{O}(G)^*$ - algebra of algebraic distributions on G, w.r.t. convolution.
Note that $\mathcal{O}(G)^*$ has a natural $\mathcal{O}(G)$-module structure.

• $\varkappa \in (\mathcal{O}(G)^* \otimes \Lambda^2 V^*)^G \rightsquigarrow$ $\boxed{\mathcal{H}_\varkappa := TV \rtimes \mathcal{O}(G)^* / ([x,y] - \varkappa(x,y) \mid x,y \in V)}$

Basic question: For which $\varkappa$, is $\mathcal{H}_\varkappa$ - a flat deformation of $\mathcal{H}_0 = SV \rtimes \mathcal{O}(G)^*$? (that is the PBW property holds).

Fact: The PBW property holds iff the Jacobi identity is satisfied:

$$\boxed{\varkappa(x,y)(z - z^g) + \varkappa(y,z)(x - x^g) + \varkappa(z,x)(y - y^g) = 0 \quad \forall x,y,z \in V, g \in G}$$

Question: Is there a good classification of distributions $\varkappa$, satisfying the Jacobi identity.

Case G-finite (due to Drinfeld '86): will be discussed on p. 4

Case G-infinite: only partial results!

③ Basic results from [EGG]

a)

- X - affine scheme of finite type $/\mathbb{C}$ $\rightsquigarrow$ $\mathcal{O}(X)$ - regular f-s $\rightsquigarrow$ $\mathcal{O}(X)^*$ - dual space.

$\mathcal{O}(X)^* \in \mathcal{O}(X)$-module: $f \in \mathcal{O}(X)$, $\mu \in \mathcal{O}(X)^* \rightsquigarrow f\mu \in \mathcal{O}(X)^* : \langle f\mu, g\rangle := \langle \mu, fg\rangle$

- $Z \subset X$ - Zariski closed $\rightsquigarrow I(Z) \subset \mathcal{O}(X)$

|| μ is supported scheme-theoretically on scheme Z if μ annihilates $I(Z)$. ($\Leftrightarrow$ can be viewed as $\in \mathcal{O}(Z)^*$)

|| μ is supported set-theoretically on Z if μ annihilates some power of $I(Z)$

Example: $\forall$ pt $a \in X \rightsquigarrow \delta_a$ is scheme-theoretic. supported at a

$\delta_a^{(n)}$ is set-theoretic. supported at a.

- $X = G$ - gp, $G \times G \to G \Longrightarrow \mathcal{O}(G)$ - coalgebra $\Rightarrow \mathcal{O}(G)^*$ - algebra.

If $G \curvearrowright Y \Rightarrow G \curvearrowright \mathcal{O}(Y), \mathcal{O}(Y)^*$. If $\mathcal{O}(Y) = \bigoplus_{V \in \mathrm{Irr}\, G} M_V \otimes V \Rightarrow \mathcal{O}(Y)^* = \prod_V M_V^* \otimes V^*$

Recall the classical result $\mathcal{O}(G) = \bigoplus_V V \otimes V^* \Rightarrow \mathcal{O}(G) = \prod_V V \otimes V^*$ as $G \times G$-mod.

For $G \curvearrowright Y$ we let Y/G denote the categorical quot, i.e. $\mathcal{O}(Y/G) = \mathcal{O}(Y)^G$.

$$(\mathcal{O}(Y)^*)^G \xleftrightarrow{1-1} \mathcal{O}(Y/G)^*$$

Notation: $C(Y) := (\mathcal{O}(Y)^*)^G (= \mathcal{O}(Y/G)^*)$

Rmk: If $Z \subset Y$ is G-inv. closed subscheme $\Rightarrow C(Z)$ - subspace of $C(Y)$.

Easy to see from case $\mu = \delta_{g_0}$ $\forall g_0 \in G$

b)

- $TV \rtimes \mathcal{O}(G)^*$ has an alg. str., with $\mu \cdot x = \sum_i v_i \cdot \langle v_i^*, gx\rangle \mu$ $\quad \forall x \in V, \mu \in \mathcal{O}(G)^*$

$\{v_i\}$ - basis of V, $\{v_i^*\}$ - dual in V^*.

Rmk: By $\langle v_i^*, gx\rangle \cdot \mu$ we mean a product of f-n $(g \mapsto \langle v_i^*, gx\rangle)$ and μ.

- $\varkappa : \Lambda^2 V \longrightarrow \mathcal{O}(G)^*$ - G-inv. pairing $\rightsquigarrow H_\varkappa := TV \rtimes \mathcal{O}(G)^* / [x,y] - \varkappa(x,y)$.

- Filtration on $H_\varkappa$: $\deg(V) = 1$, $\deg(\mathcal{O}(G)^*) = 0$

Def: PBW property holds for $H_\varkappa$ if $H_0 \twoheadrightarrow \mathrm{gr}\, H_\varkappa$ is iso.

Thm 1: PBW property $\Leftrightarrow$ Jacobi property

$$(z - z^g)\varkappa(x,y) + (y - y^g)\varkappa(z,x) + (x - x^g)\varkappa(y,z) = 0 \quad \forall x,y,z \in V.$$

▶ Algebra H_0 is Koszul. This is a general setup for equivalence of PBW with Jacobi $[\varkappa(x,y), z] + [\varkappa(y,z), x] + [\varkappa(z,x), y] = 0$

Finally: $[x, \varkappa(y,z)] = (x - x^g)\varkappa(y,z)$

We Call such alg-s $H_\varkappa$ with PBW property: continuous Hecke alg-s.

4) Main results

- **Fact 1**: If the PBW property holds for $H_\varkappa$, then $\varkappa(x,y) \in \mathcal{O}(G)^*$ is
  - set-theoretically supported at $S = \{g \in G \mid rk((1-g): V \to V) \leq 2\}$
  - supported on the scheme $G \supset \Phi = \{\Lambda^3(1-g|_V) = 0\}$.

Rmk: This should be understood as follows: $\underbrace{(V_1 - V_1^g) \wedge (V_2 - V_2^g) \wedge (V_3 - V_3^g)\,\varkappa(x,y)}_{\in \Lambda^3 V \otimes \mathcal{O}(G)^*} = 0$

- **Example 1**: For any $\tau \in (\mathcal{O}(\mathrm{Ker}\,\rho)^* \otimes \Lambda^2 V^*)^G$, $\theta \in (\mathcal{O}(\Phi)^* \otimes \Lambda^2 V^*)^G$ the distribution $\varkappa(x,y) := \tau(x,y) + \theta((1-g)x, (1-g)y)$ yields the PBW alg.

- **G-finite**: By Fact 1: $[x,y] = \tau(x,y) + \sum_{g \in S \setminus 1} \theta_g(x,y) \cdot g$, where
  $\tau \in (\mathbb{C}[\mathrm{Ker}\,\rho] \otimes \Lambda^2 V^*)^G$, θ_g - 2-form on V.
  Jacobi $\Rightarrow$ $\mathrm{Ker}(1-g|_V) \subset \mathrm{Ker}(\tau) \Rightarrow \theta_g$ - unique up to a constant. $\Rightarrow \varkappa$ is as in Ex1.
  G-inv $\Rightarrow \{\theta_g\}$ - G-inv, that is $\{\theta_g\}_{g \in C \subset S}^{\text{conj-G-class}}$ is either empty or unique up to factor
  (It exists if $rk(1-g|_V) = 2$ and centralizer $Z_g \underset{\text{triv.}}{\curvearrowright} \Lambda^2 \mathrm{Im}(1-g)$, $g \in C$)

  Thus $\varkappa(x,y) = \tau(x,y) + \sum_{g \in \text{admiss. conj. classes}} \theta_g(x,y) \cdot g$

Continuous SRA and continuous Cherednik algebra

We assume V has a G-inv. sympl form ω.

- **Example 2**: Let Σ be the closed subscheme of G defined by $p \circ \Lambda^3(1-g|_V) = 0$, where $p: \Lambda^3 V \to V$ by contracting the first 2 components using ω.
  Then for any $t \in (\mathcal{O}(\mathrm{Ker}\,\rho)^*)^G$, $c \in \mathbb{C}(\Sigma)$: $\varkappa(x,y) = \omega(x,y)t + \omega((1-g)x, (1-g)y)c$ - PBW!
  These are called continuous analogs of SRA.
  Rem: Indeed, $\Sigma = S \cup Q$, where $S = \{s \in G \mid rk(1-s|_V) \leq 2\}$, $Q = \{g \in G \mid (1-g)^2 = 0\}$
  Hence any semisimple el-t of Σ is in S. When G-finite $\Rightarrow \Sigma = S \Rightarrow$ get SRA.

- Let $V = \mathfrak{f} \oplus \mathfrak{f}^*$, $G = GL(\mathfrak{f}) \subset Sp(V)$, ω - natural pairing
  Define $\Psi \subset G$ as a closed subscheme given by $\Lambda^2(1-g|_\mathfrak{f}) = 0$.
  Obvious: $\Psi \subset \Phi$, $\{$closed pts of $\Psi\} = \{$complex refl-s, i.e. $s \in G$: $rk(1-s|_\mathfrak{f}) \leq 1\}$.

Example 3: For $t \in (\mathcal{O}(\mathrm{Ker}\,\rho)^*)^G$, $c \in \mathbb{C}(\Psi)$ define $\varkappa$ such that $\varkappa|_{\mathfrak{f} \times \mathfrak{f}} = 0 = \varkappa|_{\mathfrak{f}^* \times \mathfrak{f}^*}$, while $\varkappa(x,y) = (y,x)t + (y,(1-g)x)c$ $\forall x \in \mathfrak{f}^*, y \in \mathfrak{f}$ $\Rightarrow \varkappa$ - PBW

These are continuous Cherednik alg-s
(for G-finite, ρ - faithful repr., these are rational Cherednik alg-s)

Thm: If $\mathfrak{f}$ - faithful G-repr with $(\Lambda^2 \mathfrak{f})^G = 0$, then $\{$cont. Hecke alg$\} = \{$cont. SRA$\} = \{$cont. Cher. alg$\}$

⑤ Infinitesimal Cherednik algebras

- $\mathfrak{g} = \mathrm{Lie}(G) \Rightarrow U\mathfrak{g}$ can be viewed as subalg. of $\mathcal{O}(G)^*$ formed by distributions set-theoret. supported at $1 \in G$.

If $\varkappa$ $V\times V \longrightarrow \mathcal{O}(G)^*$ factors through $U\mathfrak{g}$ we define $H_\varkappa(\mathfrak{g}) := TV \rtimes U\mathfrak{g} / ([x,y] - \varkappa(x,y))$

$$\begin{tikzcd} V\times V \arrow[rr] \arrow[dr, dotted] & & \mathcal{O}(G)^* \\ & U\mathfrak{g} \arrow[ur] & \end{tikzcd}$$

Again define filtration by $\deg(V)=1$, $\deg(U\mathfrak{g})=0$.

! Note that $H_\varkappa = H_\varkappa(\mathfrak{g}) \underset{U\mathfrak{g}}{\otimes} \mathcal{O}(G)^*$

We call $H_\varkappa(\mathfrak{g})$ the infinit. Hecke/Cher. alg. if it satisfies PBW property.

The following theorem provides a complete description of infinitesimal Hecke alg in types A & C:

Thm 1: $H_\varkappa(\mathfrak{gl}_n)$ is PBW iff $\varkappa(x,x') = \varkappa(y,y') = 0 \quad \forall x,x' \in \mathfrak{h}^*,\ y,y' \in \mathfrak{h}$
$\varkappa(x,y) = \beta_0 \tau_0(x,y) + \beta_1 \tau_1(x,y) + \_\_$

where $\beta_i \in \mathbb{C}$, $\tau_i(x,y)$ is a symmetrization of $\alpha_i(x,y)$ which appears in expansion $(x, (1-\tau A)^{-1} y) \det(1-\tau A)^{-1}$ as a coeff. of τ^i.

Thm 2: $H_\varkappa(\mathfrak{sp}_{2n})$ is PBW iff $\varkappa(x,y) = \beta_0 \tau_0(x,y) + \beta_1 \tau_2(x,y) + \_\_$,
where $\beta_i \in \mathbb{C}$; $\tau_{2i}(x,y) = \mathrm{Sym}(\alpha_{2i})$, with $\omega(x, (1-\tau^2 A^2)^{-1} y) \det(1-\tau A)^{-1}$
$'' \sum \alpha_{2i}(x,y)\tau^{2i}$

Rmks
- $H_{a\tau_0 + b\tau_1}(\mathfrak{gl}_n) \simeq U(\mathfrak{sl}_{n+1})$ for $b \neq 0$.
- $H_{a\tau_0}(\mathfrak{sp}_{2n}) \simeq U(\mathfrak{sp}_{2n}) \ltimes W_n$
- There is no such theory, say, for O_n since in the context of continuous Hecke alg $C(\Psi)$ is 2-dim, so "no freedom".

- The proof of both thm is quite computational.
The only essential thing needed is to compute $C(\Psi)$ in group case

GL_n: $S/G \overset{s \mapsto \lambda}{\simeq} \mathbb{C}^* \Rightarrow C(S) =$ space of Fourier series $\sum_{m\in\mathbb{Z}} c_m z^m$
Ψ - reduced in this case $\Rightarrow \Psi = S \Rightarrow C(\Psi) = C(S)$

Sp_{2n} $S/G \overset{s\mapsto \lambda+\lambda^{-1}}{\simeq} \mathbb{C} \Rightarrow C(S) =$ Fourier series $\sum c_m \lambda^m$ with $c_m = c_{-m}$
$\Phi = S$ - irr. aff. alg. variety

6) Finite W-algebras

- $e\in\mathfrak{g}$ - nilpotent of a simple Lie alg $\rightsquigarrow (e,h,f)$ - Jacobson-Morozov $\mathfrak{sl}_2$-triple
- Slodowy slice: $\{e+\mathfrak{z}_{\mathfrak{g}}(f)\} =: S_e$ - transversal to the orbit $Ad(G)e$.
- Viewing $S_e\subset\mathfrak{g}^*\simeq\mathfrak{g}$ it turns out that S_e inherits the Poisson str. of $\mathfrak{g}^*$.
- Premet, Gan - Ginzburg: quantizations of Poisson alg. $\mathcal{O}(S_e)$.

W-algebra

$\mathfrak{g}=\bigoplus_{i\in\mathbb{Z}}\mathfrak{g}(i)$ w.r.t. $ad(h)$ $\rightsquigarrow$ $\mathfrak{m}:=\bigoplus_{i\le -2}\mathfrak{g}(i)\oplus\mathfrak{l}$, where $\mathfrak{l}\subset\mathfrak{g}(-1)$ is a Lagrangian of $(\mathfrak{g}(-1),\omega)$ $\omega(\chi,\eta):=\langle e,[\chi,\eta]\rangle$.

$\rightsquigarrow \mathfrak{m}':=\{\xi-\langle e,\xi\rangle \mid \xi\in\mathfrak{m}\}$

$\boxed{U(\mathfrak{g},e):=\left(U(\mathfrak{g})/U(\mathfrak{g})\mathfrak{m}'\right)^{ad\,\mathfrak{m}}}$ - W-alg with multipl. induced from $U\mathfrak{g}$.

Kazhdan filtration: $F_k U(\mathfrak{g}):=\sum_{i+2j\le k}\left(F_j^{PBW}U(\mathfrak{g})\cap U(\mathfrak{g})(i)\right)$

$\rightsquigarrow$

$\{F_k U(\mathfrak{g},e)\}$ - induced filtration.

First main result: $gr_{F_\bullet}U(\mathfrak{g},e)\simeq\mathcal{O}(S_e)$ as Poisson alg-s

The theory of these has been extensively studied recently by Losev, Brundan - Kleshchev, Premet, ..

Rmks:

1. $(U\mathfrak{g})^G\simeq\mathfrak{z}(U\mathfrak{g})\xrightarrow{\sim}\mathfrak{z}(U(\mathfrak{g},e))$
2. There is a natural action of $Z(e,h,f)$ ↻ $U(\mathfrak{g},e)$.
3. It is used in the proof of our main result:

S_e is a universal Poisson deformation of $S_e^\circ:=S_e\cap\mathcal{N}$

$$\begin{array}{l}\mathfrak{g}\to\mathfrak{g}/\!/G\simeq\mathfrak{h}/W\\ \cup\\ S_e\end{array}\quad\rightsquigarrow\quad\begin{tikzcd} S_e^\circ \arrow[d] & S_e \arrow[d] \\ \{0\} \arrow[r, hook] & \mathfrak{g}/\!/G \end{tikzcd}\qquad\begin{tikzcd} S_e \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \mathcal{X} \arrow[d, "\rho"] \\ \mathfrak{g}/\!/G & T\ni t \arrow[l, dashed] \end{tikzcd}\qquad\begin{array}{l}\mathcal{X}\text{ - Poisson}/T\\ \mathcal{X}_0:=\rho^{-1}(t)\simeq S_e^\circ\end{array}$$

[Lehn - Namikawa - Sorger]

7 Main Result

- **Observation #1**. Need to consider not $\mathcal{H}_\chi(\mathfrak{g})$, but $H_m(\mathfrak{g})$, where β_i - formal variables

In $\mathfrak{g} = \mathfrak{gl}_n$: $H_m(\mathfrak{gl}_n) = \mathcal{U}(\mathfrak{gl}_n) \ltimes T(V_n \oplus V_n^*)[\zeta_0, \to \zeta_{m-2}] / [x, y] = \zeta_0 z_0 + \_ + \zeta_{m-2} z_{m-2} + z_m$

$\mathfrak{g} = \mathfrak{sp}_{2n}$: $H_m(\mathfrak{sp}_{2n}) = \mathcal{U}(\mathfrak{sp}_{2n}) \ltimes T(V_{2n})[\zeta_0, \to \zeta_{m-1}] / [x, y] = \zeta_0 z_0 + \_ + \zeta_{m-1} z_{m-1} + z_m$

- **Notation**: $e_m \in \mathfrak{sl}_n$ is a Jordan type $(1^{n-m}, m)$ nilp.

$$e_m = \begin{pmatrix} 0 & 0 \\ 0 & \begin{smallmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{smallmatrix} \end{pmatrix} \begin{matrix} \}\, n-m \\ \}\, m \end{matrix} \rightsquigarrow f_m = \begin{pmatrix} 0 & \begin{smallmatrix} & & \\ m-1 & 0 & \\ 0 & 2(m-2) \ddots & 0 \end{smallmatrix} & 0 \end{pmatrix} \}\, m$$
$$\underbrace{\qquad\qquad}_{m}$$

In case of $\mathfrak{g} = \mathfrak{sp}_{2n}$ e_m denotes nilp. of Jordan type $(1^{2n-2m}, 2m)$

$$\begin{pmatrix} \begin{smallmatrix} 0 & 0 \\ & 0 \end{smallmatrix} & 0 \\ 0 & \begin{smallmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ & & \ddots \\ & & & 0 \end{smallmatrix} \end{pmatrix}$$

where sympl. form is via $J = \begin{pmatrix} 0 & & & 1 \\ & & -1 & \\ & 1 & & \\ -1 & & & 0 \end{pmatrix}$

- **Main thm**:
$$H_m(\mathfrak{gl}_n) \simeq \mathcal{U}(\mathfrak{sl}_{n+m}, e_m)$$
$$H_m(\mathfrak{sp}_{2n}) \simeq \mathcal{U}(\mathfrak{sp}_{2n+2m}, e_m).$$

Sketch the proof: Indicate at the Poisson level the corresponding parts of $\mathfrak{g}$ & V.

8 Consequences

1. The center of $H_m(gl_n)$ and $H_m(Sp_{2n})$ is a polyn. alg. in generators ζ_i and n more generators!

   [Tik]: $\mathfrak{z}(H_x(gl_n)) = k[t_1+c_1, \ldots, t_n+c_n]$ for some $c_i \in \mathfrak{z}(U_{\mathfrak{g}})$,

   where $t_i := \sum x_j[\beta_i, y_j]$, in particular, $t_1 = \sum x_j y_j$.

2. # Sympl. leaves of the full central reduction is finite.

3. Analogue of the Kostant's thm:

   (a) $H_x(\mathfrak{g})$ is free over $\mathfrak{z}(H_x(\mathfrak{g}))$

   (b) Full central reduction of $gr\, H_x(\mathfrak{g})$ is a normal, complete intersection, integral domain.

4. Classification of fin. dim. repr. of $H_x(gl_n)$

   $H_x(gl_n) \searrow$ W-alg

   [DT] $\xleftrightarrow{\text{agree}}$ Brundan - Kleshchev in general setup

5. There is also a similar f-la for the Shapovalov determinant.

Completions

$$H_{\hbar,m}(gl_n)^{\wedge e_n} \simeq H^{\wedge 0}_{\hbar,m+1}(gl_{n-1}) \hat{\otimes}_{\mathbb{C}[[\hbar]]} W_{\hbar,n}^{\wedge e_n} \qquad H_{\hbar,m}(Sp_{2n})^{\wedge e_{2n}} \simeq H_{\hbar,m+1}(Sp_{2n-2})^{\wedge 0} \hat{\otimes}_{\mathbb{C}[[\hbar]]} W_{\hbar,2n}^{\wedge e_{2n}}$$

Comments:

- $H_{\hbar,m}(\mathfrak{g}) := Rees_\hbar(H_m(\mathfrak{g}))$
- $W_{\hbar,n} = Rees_\hbar(Weyl) = \mathbb{C}\langle x_1, \ldots, x_n; \partial_1, \ldots, \partial_n\rangle[\hbar] / [\partial_i, x_j] - \hbar^2 \delta_i^j$
- The above completion is a Losev's technique similar to usual completions in comm. algebra.

  In general, this is defined as follows:

  - Let Y be an affine Poisson scheme, $\mathbb{C}^* \curvearrowright Y$, s.t. $\deg \{,\} = -2$
  - Let $\mathcal{A}_\hbar$ be a flat graded $\mathbb{C}[\hbar]$-alg, $\deg(\hbar)=1$, s.t. $\mathcal{A}_\hbar/(\hbar) \simeq \mathbb{C}[Y]$ graded Poisson
  - Pick $x \in Y \rightsquigarrow I_x \subset \mathbb{C}[Y] \rightsquigarrow \tilde{I}_x \subset \mathcal{A}_\hbar$ - inverse image

  $\mathcal{A}_\hbar^{\wedge x} := \varprojlim \mathcal{A}_\hbar / \tilde{I}_x^n$

  $\uparrow$ complete topol. $\mathbb{C}[[\hbar]]$-alg., s.t. $\mathcal{A}_\hbar^{\wedge x}/(\hbar) = \mathbb{C}[Y]^{\wedge x}$

Above isomorphisms are analogous to Bezrukavnikov - Etingof isomorphisms.

9 Final comments

1. Explanation why $\mathfrak{z}(\mathcal{U}\mathfrak{g}) \longrightarrow \mathfrak{z}(\mathcal{U}(\mathfrak{g},e))$ is iso.

$\hookleftarrow$: On the level of associated graded $gr(\mathfrak{z}(\mathcal{U}\mathfrak{g})) = (S\mathfrak{g})^G$ – Poisson center

So when restricted to Slodowy slice it produces f-n on S_e

If $f \in (S\mathfrak{g})^G$ is s.t. $f|_{S_e} = 0 \Rightarrow f \equiv 0$ as Slodowy slice transversal to Ge.

$\twoheadrightarrow$: Consider the Chevalley map $\mathfrak{g} > S_e \overset{\pi}{\twoheadrightarrow} \mathfrak{g}/\!\!/G$

Any central element $z \in \mathfrak{z}(\mathcal{U}(\mathfrak{g},e)) \rightsquigarrow gr\, z \in \mathfrak{z}.gr\, \mathcal{U}(\mathfrak{g},e)) = \mathfrak{z}_{Pois}(\mathbb{C}[S_e])$

Premet proved: all scheme theoretic fibers of π – irred, reduced & $gr\, \mathcal{U}(\mathfrak{g},e)$/flat$(S\mathfrak{g})^G$

Each fibre of π has only finitely many Poisson leaves, $gr\, z$ is constant on each Poisson leaf $\Rightarrow gr\, z$ is constant on fibres of π (as they are irreduc.)

$S_e \overset{\pi}{\twoheadrightarrow} \mathfrak{g}/\!\!/G$ $\qquad gr\, z \in \mathbb{C}[S]$, it is constant along fibres $\rightarrow$ pull-back of el-t of $\mathbb{C}[\mathfrak{g}/\!\!/G]$

Done!

2. Bezrukavnikov – Etingof theory (for rational Cherednik alg-s)

W – reflection gp, $\rho: W \to GL(\mathfrak{h})$, $b \in \mathfrak{h} \rightsquigarrow \underline{W} := W_b < W$ – also reflection gp

Reflections $\underline{S}$ in $\underline{W}$ are just $S \cap \underline{W} \Rightarrow c: S \to \mathbb{C}$ induces $\underline{c}: \underline{S} \to \mathbb{C}$.

They produced functor

$$\mathcal{O}_c(W, \mathfrak{h}) \underset{Ind}{\overset{Res}{\rightleftarrows}} \mathcal{O}_{\underline{c}}(\underline{W}, \mathfrak{h}_{\underline{W}})$$

unique W-stable complement to $\mathfrak{h}^{\underline{W}}$.

Main tool they use: isomorphism $H_c(W,\mathfrak{h})^{\wedge b} \simeq Z(W, \underline{W}, \underline{H}_c^{\wedge b})$

$\mathbb{C}[\mathfrak{h}/\underline{W}]^{\wedge b} \otimes_{\mathbb{C}[\mathfrak{h}/\underline{W}]} H_c(\underline{W},\mathfrak{h})$